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Let $\sqrt{(r^2+4a^2)}=2y-r$, $\frac{a}{2}(\sqrt{5}+1)=y'$.

$$\therefore\ \Delta=\tfrac{1}{6}\pi a^2-\tfrac{1}{3}a^2\sin^{-1}(\sqrt{5}-2)+\frac{\sqrt{a}}{3a}\int_a^{y'}\frac{(3y^2+3a^2+2ay)(y-a)^3dy}{y^3\sqrt{y}}$$

$$=\tfrac{1}{6}\pi a^2-\tfrac{1}{3}a^2\sin^{-1}(\sqrt{5}-2)+\frac{\sqrt{a}}{3a}\int_a^{y'}(3y\sqrt{y}-7a\sqrt{y}+6a^2/\sqrt{y}-6a^3/y\sqrt{y}$$

$$+7a^4/y^2\sqrt{y}-3a^5/y^3\sqrt{y})dy=\tfrac{1}{6}\pi a^2-\tfrac{1}{3}a^2\sin^{-1}(\sqrt{5}-2)+\tfrac{2}{45}a^2[(86-13\sqrt{5})$$

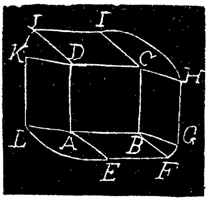
$$\sqrt{\frac{\sqrt{5}+1}{2}}-128+11(11-2\sqrt{5})\sqrt{\frac{\sqrt{5}-1}{2}}].$$

Solved with a different result by the *PROPOSER*.

134. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

An ellipse, semi-axes a, b, is placed on a square, side c. Find the chance that center of ellipse is on the square.

Solution by the PROPOSER.



Let $ABCD$ be the square, and let the ellipse move parallel to itself so that the major axis makes an angle θ with the side BC. Then the center of the ellipse will describe the area $GHIJKLEFG$. Let $(a^2-b^2)/a^2=e^2$, p=chance. The perpendicular distance from BC to $GH=a\sqrt{(1-e^2\cos^2\theta)}$, the perpendicular distance from DC to $IJ=a\sqrt{(1-e^2\sin^2\theta)}$.

Area$=\pi ab+c^2+2ac\,[\sqrt{(1-e^2\sin^2\theta)}+\sqrt{(1-e^2\cos^2\theta)}]=u$.

$$1/p=\int_0^{\frac{1}{2}\pi}ud\theta\Big/\int_0^{2\pi}c^2d\theta=\frac{2}{\pi c^2}\int_0^{\frac{1}{2}\pi}ud\theta.\quad\therefore\ 1/p=[\pi ab+c^2+\frac{8ac}{\pi}E(e,\tfrac{1}{2}\pi)]/c^2.$$

If $a=b$, $1/p=(\pi a^2+c^2+4ac)/c^2$. If $a=b=c$, $1/p=\pi+1+4$.

135. Proposed by LON C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

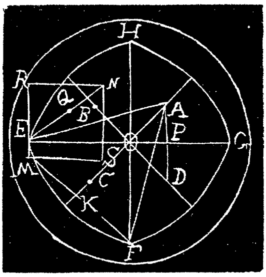
If the line joining two points taken at random in the surface of a given circle be the diagonal of a square, the chance that the square lies wholly within the circle is $2-4/\pi$.

Solution by the PROPOSER.

Let MN be the line joining the random points M, N; $MRNS$ the square; Q, its center; O, the center of the circle. Let the square move about the circle, so as to be within it, but in contact with it, the diagonal MN remaining parallel to itself; Q will describe the figure $EFGH$, whose boundary consists of the four equal arcs, EF, FG, GH, HE, whose centers are A, B, C, D, and radius equal to that of the given circle.

Now while MN is given in length and direction, the area of the figure $EFGH$ represents the number of ways the two points can be taken, so that the square will be wholly within the circle.

Let $AE=r$, $MN=2x$, $\angle AEP=\theta$, $\phi=$the angle which MN makes with some fixed line, area $EFGH=u$.



Then we have $OA=x$, $AP=\frac{1}{2}x\sqrt{2}$, $\angle EAK=\angle AOP-\angle AEP=\frac{1}{4}\pi-\theta$, $\frac{1}{2}x\sqrt{2}=r\sin\theta$, $EK=r\sin(\frac{1}{4}\pi-\theta)$, area $EOF=r^2(\frac{1}{4}\pi-\theta)-rx\sin(\frac{1}{4}\pi-\theta)=r^2(\frac{1}{4}\pi-\theta-\sin\theta\cos\theta+\sin^2\theta)$, and $u=r^2(\pi-4\theta-4\sin\theta\cos\theta+4\sin^2\theta)$. An element of the circle at the point N is $4xdxd\phi$, or $8r^2\sin\theta\cos\theta d\theta d\phi$. The limits of θ are 0 and $\frac{1}{4}\pi$; and of ϕ, 0 and 2π. Hence the chance in question is

$$\delta=\frac{1}{\pi^2r^4}\int_0^{\frac{1}{4}\pi}\int_0^{2\pi}r^2(\pi-4\theta-4\sin\theta\cos\theta+\sin^2\theta).8r^2\sin\theta\text{cas}\theta d\theta d\phi$$

$$=\frac{16}{\pi}\int_0^{\frac{1}{4}\pi}(\pi-4\theta-4\sin\theta\cos\theta+4\sin^2\theta)\sin\theta\cos\theta d\theta=2-(4/\pi).$$

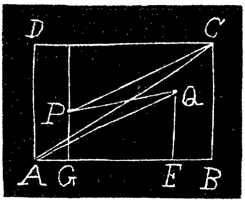
An excellent solution of this problem was also received from *G. B. M. ZERR.*

136. **Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.**

By direct computation find the average distance between two points in the surface of a given rectangle, but on the opposite sides of a diagonal.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and the PROPOSER.

Let P, Q be the random points, $AB=a$, $BC=b$, $a^2+b^2=d^2$, $AE=x$, $QE=y$, $CF=u$, $PF=v$. Then $PQ=\sqrt{[(x+u-a)^2+(y+v-b)^2]}$. The limits of x are 0 and a; of u, 0 and a; of y, 0 and bx/a; of v, 0 and bu/a. Let $(x+u-a)=c$, $\triangle=$average distance.



$$\therefore\triangle=\frac{4}{a^2b^2}\int_0^a\int_0^a\int_0^{bx/a}\int_0^{bu/a}\sqrt{[c^2+(y+v-b)^2]}$$

$$\times dxdudydv=\frac{2}{a^2b^2}\int_0^a\int_0^a\int_0^{bx/a}\Big[\Big(\frac{ay+bu-ab}{a^2}\Big)$$

$$\times\sqrt{(a^2c^2+(ay+bu-ab)^2]}-(y-b)\sqrt{[c^2+(y-b)^2]}$$

$$+c^2\log\Big(\frac{ay+bu-ab+\sqrt{[a^2c^2+(ay+bu-ab)^2)}}{ay-ab+a\sqrt{[c^2+(y-b)^2]}}\Big)\Big]dxdudy$$

$$=\frac{2}{3a^5b^2}\int_0^a\int_0^a\{[2a^2c^2-b^2(u-a)^2]\sqrt{[a^2c^2+b^2(u-a)^2]}+[(2a^2c^2-b^2)(x-a)^2]$$